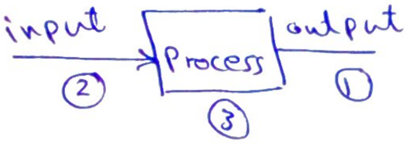
24/11/2015 الثلاثاء م.شهاب محاضرة 8

Control Systems Design :-

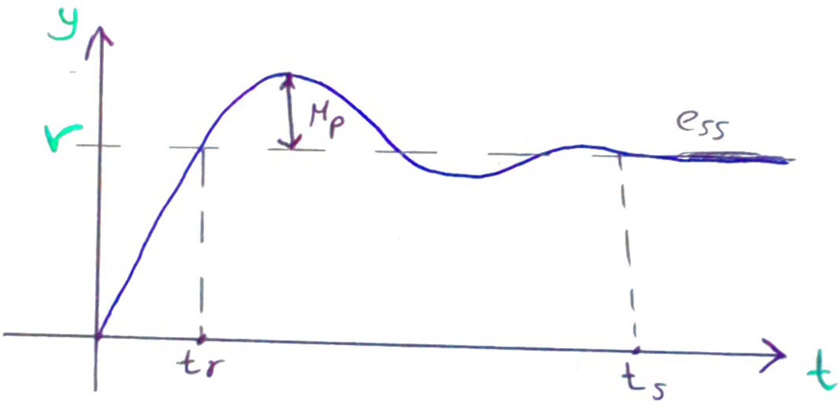
Engineering Problems

- Analysis Problem:

- Design Problem:



Control Systems Design Specs :-



$$t_r = \frac{\pi - \cos^{-1}\zeta}{\omega_n\sqrt{1-\zeta^2}} \quad ; \quad t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$$

$$M_p = e^{-\zeta\pi/\sqrt{1-\zeta^2}} \quad ; \quad t_s = \frac{4}{\zeta\omega_n}$$

e_{ss} :

type 0: $e_{ss} = \dfrac{1}{1+K_p}$; type 1: $e_{ss} = \dfrac{1}{K_v}$

type 2: $e_{ss} = \dfrac{1}{K_a}$

$$K_p = \lim_{s \to 0} GH(s) \quad ; \quad K_v = \lim_{s \to 0} s\,GH(s)$$

$$K_a = \lim_{s \to 0} s^2\,GH(s)$$

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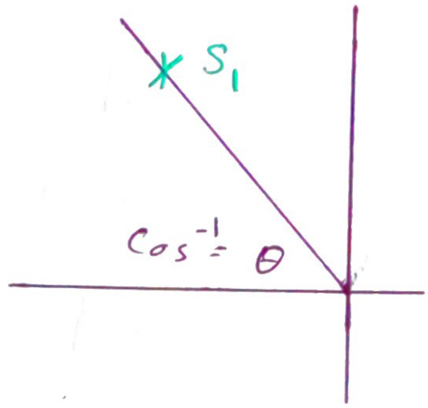
* Design Procedure

(1) desired C.L. c/c eqn

$$S_{1,2} = \underbrace{-\zeta\omega_n}_{\sigma} \pm \underbrace{\omega_n\sqrt{1-\zeta^2}}_{\omega_d}$$



- Control Design Objective :-
Add dynamic Components to the system to change the overall System dynamics i.e. Change the Locations of C.L. poles.

Remember what we've done in practice :-

* Requirements
* Modelling of system dynamics (Analysis) ⇒ (Dynamics don't meet desired specs)
* Controller (Add Components)

Response

Transient
M_p, t_r, t_s

Steady state
e_{ss}

* Transient Response Compensation

→ Lead Compensator

→ PD

* Steady State error Compensation

→ Lag Compensator

→ PI

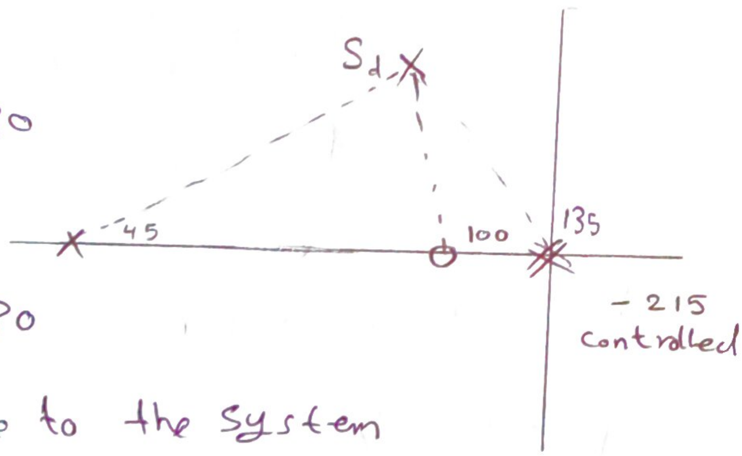
* For overall Compensation

→ Lead - Lag Compensator

→ PID

Lead Compensator :-

at $S_d : \sum \phi_z - \sum \phi_p \neq \pm 180$



desired

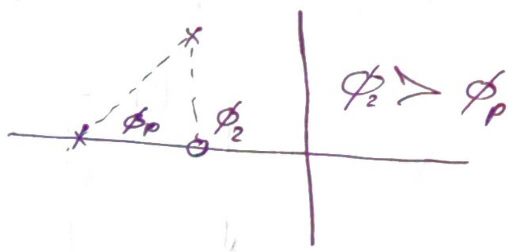
at $S_d : \sum \phi_z - \sum \phi_p = \pm 180$

By Adding a zero and a pole to the system

$$\sum \phi_{z_c} - \sum \phi_{p_c} = -180 - \phi$$

$$\boxed{\angle GH(s) + \phi_c = -180}$$

Root Locus Angle Condition



* Lag Compensator design

① Calculate $\beta = \dfrac{K_c}{\underbrace{K_{un}}_{\text{uncontrolled}}} * 1.1 \rightarrow$ Safety factor

K_c: desired value of DC gain

K_{un}: the system value of DC gain

② Assume zero location (Z_c) in the range of 10% from the 2nd dominent pole of the system:

$$Z_c = \frac{10}{100} P_2 \Leftarrow \text{second dominant pole}$$

$$P_c = \frac{Z_c}{\beta} \qquad \underbrace{K_{dc}}_{\text{of controller}} = \frac{Z_c}{Z_c/\beta} = \beta$$

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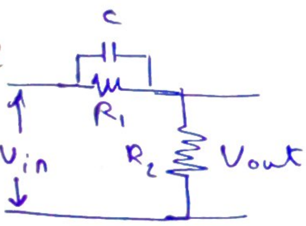
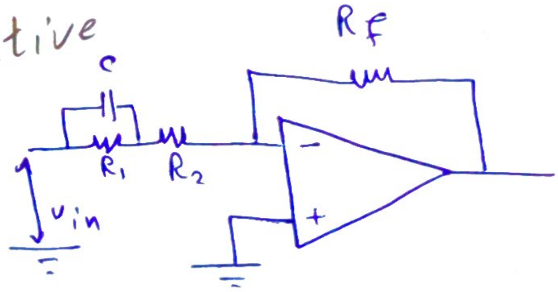
* Lead Compensator

Passive



Active



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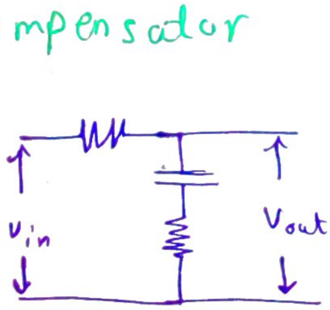
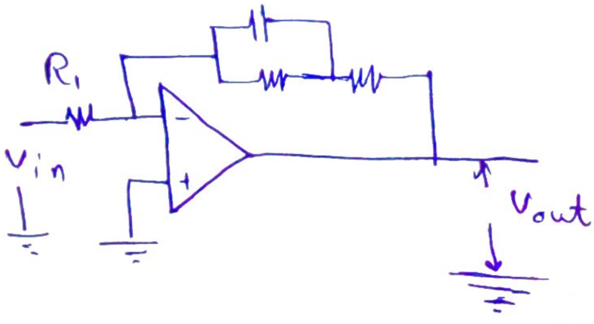
* Lag Compensator

Passive





Example :-

A type 1 system has an O.L.T.F $GH(s) = \frac{K}{s(s+1)(s+4)}$
It is desired to compensate such system to match the following design specs:-

$\zeta = 1/2$ $\qquad e_{ss} \leq 0.1$

$\omega_n = 2$ rad/se

1. desired locations for closed loop poles

$$S_{1,2} = -\zeta\omega_n \pm \omega_n \sqrt{1-\zeta^2}$$
$$= -1 \pm j\sqrt{3}$$

2. from the angle condition

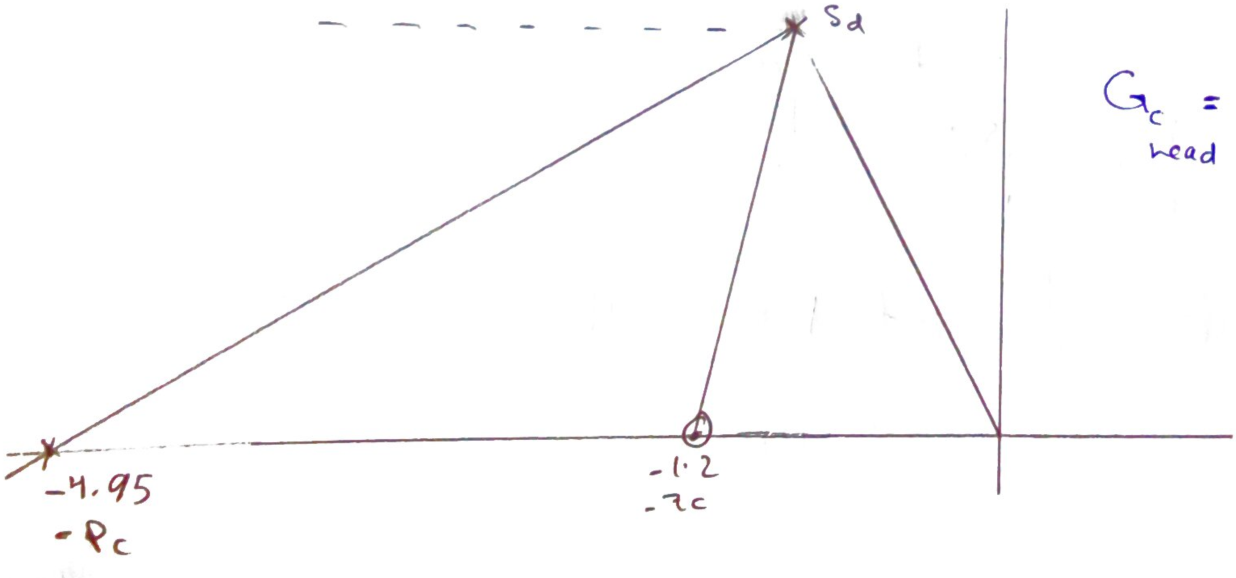
$$\angle GH(s) + \phi_c = -180$$

$$\angle \frac{K}{s(s+1)(s+4)}\Big|_{s=-1+j\sqrt{3}} = 0 - \angle -1+j\sqrt{3} - \angle j\sqrt{3} - \angle 3+j\sqrt{3} + \phi_c = -180$$

$$-120 - 90 - 30 + \phi_c = -180$$

$$\Rightarrow \phi_c = 60$$

3. Determine the Locations of Z_c & P_c



$G_{c\,\text{lead}} = \frac{S+1.2}{S+4.95}$

The total T.F.

$$GH(s) = \frac{K(s+1.2)}{s(s+1)(s+4)(s+4.95)}$$

4. Determine K from Magnitude condition

$$\| K\, GH(s) \| = 1$$

$$K = \frac{1}{\|GH(s)\|} = \left. \frac{\|s\| \, \|s+1\| \, \|s+4\| \, \|s+4.95\|}{\|s+1.2\|} \right|_{s=-1+j\sqrt{3}} = 30$$

5. Check steady state error

type 1 system $\Rightarrow e_{ss} = \frac{1}{K_v}$

$$K_v = \lim_{s \to 0} s\,GH(s)$$

$$K_v = \lim \frac{s(s+1.2) * 30}{s(s+1)(s+4)(s+4.95)} = 1.81$$

$$e_{ss} = \frac{1}{K_v} = 0.55 \quad ; \quad \boxed{e_{ss} \leqslant 0.1}$$

Required

to reduce e_{ss} Add Lag compensator

desired $e_{ss} = 0.1 \Rightarrow K_v = \frac{1}{e_{ss}} = 10$

* Lag compensator design

1. $K_{un} = 1.82$

$K_c = 10$

$$\beta = \frac{10}{1.82} * 1.1 = 6.04$$

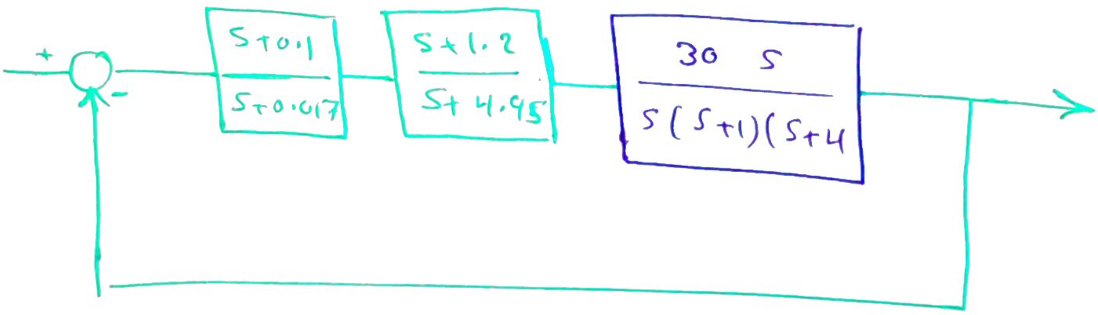
② Locations of Z_c and P_c

$$Z_c = \frac{10}{100} P_2 = 0.1 * -1 = -0.1$$

$$P_c = \frac{Z_c}{\beta} = \frac{-0.1}{6.04} = -0.017$$

$$G_{c_{Lag}} = \frac{s+0.1}{s+0.017}$$

$$GH(s) = \frac{30(s+0.1)(s+1.2)}{s(s+0.017)(s+1)(s+4)(s+4.95)}$$



Check e_{ss}

$$\lim_{s \to 0} s\,GH(s) = \frac{30\cancel{s}(s+0.1)(s+1.2)}{\cancel{s}(s+0.017)(s+1)(s+4)(s+4.95)} \; ; e_{ss} = 0.09$$